9.2b Taylor Series

Maclaurin series you should have memorized: p 491

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!}(-1)^n \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!}(-1)^n$$

n starts at 0

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^n}{n}(-1)^{n+1} \dots$$

n starts at 1

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p 491

$$10.\ f(x) = x^2 \cos x = x^2 - \frac{x^4}{2!} + \frac{x^6}{4!} \dots \frac{x^{2n+2}}{(2n)!}(-1)^n \dots$$

$$x^2 \cos x = x^2\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots \frac{x^{2n}}{(2n)!}(-1)^n \dots\right)$$

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Taylor series for f(x) centered on x=a

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

$$\dots + \frac{f^{(n)}(a)}{n!}(x-a)^n \dots$$

approximates f(x) for values of x close to a

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Find the Taylor series generated by f(x)=e^x at x=2 $\{a=2\}$

$f = e^x \quad f(2) = e^2$

$f' = e^x \quad f'(2) = e^2$

$f'' = e^x \quad f''(2) = e^2$

$\vdots$

e^2

$$e^x = e^2 + e^2(x-2) + \frac{e^2}{2!}(x-2)^2 + \frac{e^2}{3!}(x-2)^3 + \dots \frac{e^2}{n!}(x-2)^n$$

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Find the Maclaurin series for $f(x)=\frac{1+\cos(2x)}{2}$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{x^{2n}}{(2n)!}(-1)^n \cdots$$

$$\frac{1+\cos(2x)}{2} = \frac{1}{2}+\frac{1}{2} - \frac{(2x)^2}{2\cdot 2!} + \frac{(2x)^4}{2\cdot 4!} \cdots \frac{(2x)^{2n}}{2\cdot(2n)!}(-1)^n \cdots$$

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Find the first four nonzero terms for the Maclaurin series for $\sin(x^2)$

Use this series to find a corresponding series for $\int \sin(x^2)\,dx$

(why not just evaluate $\int \sin(x^2)\,dx$?)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!}(-1)^n \cdots$$

$$\sin(x^2) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} \cdots + \frac{(x^2)^{2n+1}}{(2n+1)!}(-1)^n \cdots$$

$$\int \sin(x^2)\,dx = \int x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} \cdots + \frac{x^{4n+2}}{(2n+1)!}(-1)^n\,dx \cdots$$

$$\int \sin(x^2)\,dx = \frac{x^3}{3} - \frac{x^7}{7\cdot 3!} + \frac{x^{11}}{11\cdot 5!} \cdots \frac{x^{4n+3}}{(4n+3)(2n+1)!}(-1)^n \cdots$$

n starts at 0

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$\sqrt{2}$ $\cancel{\frac{p}{q}}$ 1.141...

$$\sqrt{2} = 1 + \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} \cdots$$

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